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Vibrational convection in a horizontal fluid layer with internal heat sources

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Abstract—A theoretical examination is made of the convection of a fluid with uniform internal heat sources in a plane horizontal layer in the presence of harmonic high-frequency vibration of arbitrary orientation. The solid boundaries of the layer are kept at identical temperatures. The linear problem of mechanical equilibrium stability is solved numerically. The stability limits, parameters and form of critical disturbances are determined. The non-linear finite-amplitude regimes of convection and heat transfer are investigated by the method of finite differences. The presence of stationary and wave (drifting) regimes is established and their characteristics are studied.

1. INTRODUCTION

THERE HAS recently been considerable interest shown in the study of the sensitivity of heat convection and convective stability of a fluid to various complicating factors such as rotation, nonhomogeneity of composition, magnetic field, forced flow and many others, with vibration being among them. It is shown in theoretical and experimental studies of many authors that vibrations can substantially enhance convective heat transfer. In the case when mechanical equilibrium of a non-uniformly heated fluid is possible, vibration of a fluid filled cavity exerts a strong stabilizing or destabilizing effect leading, in particular, to the parametrical resonance-type phenomena [1]. A study of the influence of vibrations on the stability of convective flows has also been initiated [2–6]. The specifics of the vibrational effect are evidenced by the fact that under certain conditions it can generate convective flows by itself even in the absence of the static gravity field, i.e. under the conditions of zero gravity. This testifies to the presence of some kind of special vibrational heat convection mechanism [7, 8]. Studies on vibrational convection in zero gravity are surveyed in ref. [9].

In the present paper the solution of a new problem of the vibrational convection theory is attempted. The convective motion of a fluid with internal heat sources uniformly distributed over its volume is considered. The fluid fills a plane horizontal layer which has boundaries of identical temperatures and which vibrates in some arbitrary direction. First it is shown that under these conditions the fluid can be in the state of mechanical equilibrium. Then, within the framework of the linear theory of small disturbances, the stability of mechanical equilibrium is studied, the limits of stability and the characteristics of critical disturbances are determined, including their structure. Finally, stationary and non-stationary finite-amplitude regimes of vibrational convection developing due to the loss of stability equilibrium are investigated and their hydrodynamic and heat transfer characteristics are determined.

2. FORMULATION OF THE PROBLEM. BASIC EQUATIONS

A plane infinite horizontal fluid layer is confined between two isothermal planes $z = 0$ and h (Fig. 1) which are kept at the same temperature and taken as the reference point. Internal heat sources of strength Q are uniformly distributed over the fluid volume. The fluid layer with its boundaries undergoes linear harmonic vibration with displacement amplitude b and angular frequency Ω of some arbitrary orientation characterized by the unit vector $\mathbf{n}$ inclined at an angle α to the horizontal x-axis.

The vibration frequency is assumed to be large enough so that its period is much smaller than all the other characteristic hydrodynamic times. By representing the fluid velocity, temperature and pressure as a sum of mean parts slowly changing in time and of quickly oscillating components, it is possible, following the averaging procedure, to obtain a closed system of equations for mean fields from the Oberbeck–Boussinesq equations [10, 11]. The unknowns in this system are the mean velocity $\mathbf{v}$, temperature T, pressure p and a new unknown function $\mathbf{w}$ which is the solenoidal part of the vector field $T\mathbf{n}$ (this function

NOMENCLATURE

- b amplitude of vibration
- c phase velocity
- F stream function of $\mathbf{w}$ field
- g gravity acceleration
- h thickness of layer
- k wave number
- L spatial period
- $\mathbf{n}$ unit vector along the axis of vibration
- Nu Nusselt number
- p pressure
- Pr Prandtl number
- Q power of heat output
- Ra_q Rayleigh number
- Ra_v vibrational Rayleigh number
- T temperature
- U drift velocity
- $\mathbf{v}(v_x, v_y, v_z)$ velocity
- $\mathbf{w}(w_x, w_y, w_z)$ solenoidal part of $T\mathbf{n}$ field
- (v, w) amplitudes of disturbances
- (x, y, z) Cartesian coordinates.

Greek symbols
- α angle between the axis of vibration and the x-axis
- β coefficient of thermal expansion
- γ unit vector along the z-axis
- ε dimensional vibrational parameter, $\frac{1}{2}(\beta b\Omega)^2$
- θ amplitude of temperature disturbance
- λ decrement, $\lambda_r + i\lambda_i$
- ν kinematic viscosity
- ρ density
- χ thermal diffusivity
- ψ stream function of $\mathbf{v}$ field
- Ω angular frequency of vibration.

Subscripts
- m extreme value
- 0 equilibrium solution
- * critical value.

Superscript
- $'$ disturbance.

is proportional to the oscillating velocity component amplitude in the proper system of coordinates). The system of equations is

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu\Delta\mathbf{v} + g\beta T\gamma + \varepsilon(\mathbf{w}\nabla)(T\mathbf{n} - \mathbf{w}) \quad (1)$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\nabla T = \chi\Delta T + \frac{Q}{\rho c_p} \quad (2)$$

$$\operatorname{div}\mathbf{v} = 0 \quad (3)$$

$$T\mathbf{n} = \mathbf{w} + \nabla\varphi \quad (4)$$

$$\operatorname{div}\mathbf{w} = 0. \quad (5)$$

On the solid boundaries of the layer the non-slip and isothermicity conditions and the condition of non-overflowing for the oscillating velocity should be imposed

$$z = 0, h: \quad \mathbf{v} = 0, \quad T = 0, \quad w_z = 0. \quad (6)$$

Boundary conditions (6) are supplemented with the closure for the mean and oscillating velocity components, i.e. the zero flow rate of the fluid through the layer cross-section.

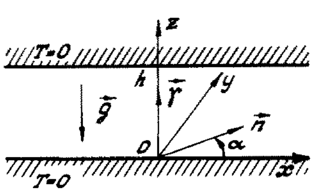

FIG. 1. Geometry of the problem.

The equations and boundary conditions can be written in dimensionless form with the following units: h for length, h^2/ν for time, χ/h for velocity, $Qh^2/\rho c_p\chi$ for temperature, and $\rho\nu\chi/h^2$ for pressure. With the overbar indicating dimensionless quantities the following relations are obtained:

$$x = h\bar{x}, \quad t = (h^2/\nu)\bar{t}, \quad \mathbf{v} = (\chi/h)\bar{\mathbf{v}},$$

$$T = (Qh^2/\rho c_p\chi)\bar{T}, \quad \bar{\mathbf{w}} = (Qh^2/\rho c_p\chi)\bar{\mathbf{w}},$$

$$p = (\rho\nu\chi/h^2)\bar{p}.$$

With the overbar omitted, the dimensionless equations and boundary conditions can be written as

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{Pr}(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \Delta\mathbf{v} + Ra_q T\gamma + Ra_v(\mathbf{w}\nabla)(T\mathbf{n} - \mathbf{w}) \quad (7)$$

$$Pr\frac{\partial T}{\partial t} + \mathbf{v}\nabla T = \Delta T + 1 \quad (8)$$

$$\operatorname{div}\mathbf{v} = 0 \quad (9)$$

$$T\mathbf{n} = \mathbf{w} + \nabla\varphi \quad (10)$$

$$\operatorname{div}\mathbf{w} = 0. \quad (11)$$

The boundary conditions and closure conditions are

$$z = 0, 1: \quad \mathbf{v} = 0, \quad T = 0, \quad w_z = 0 \quad (12)$$

$$\int_0^1 v_x\,dz = 0, \quad \int_0^1 v_y\,dz = 0 \tag{13a}$$

$$\int_0^1 w_x\,dz = 0, \quad \int_0^1 w_y\,dz = 0. \tag{13b}$$

The problem contains three dimensionless parameters: the Rayleigh number, its vibrational analogue and the Prandtl number

$$Ra_q = g\beta Qh^5/\rho c_p \nu\chi^2 \tag{14a}$$

$$Ra_v = (1/2\nu\chi^3)(\beta b\Omega Qh^3/\rho c_p)^2 \tag{14b}$$

$$Pr = \nu/\chi. \tag{14c}$$

3. THE MECHANICAL QUASI-EQUILIBRIUM

A non-uniformly heated fluid in a static gravity field in the presence of high-frequency vibration can be found in the mechanical quasi-equilibrium state when the mean flow is absent and the oscillating velocity is different, generally speaking, from zero. The equilibrium conditions result from the general equations of vibrational convection, equations (7)–(11), by assuming $\mathbf{v} = 0$ and the other mean quantities to be stationary. For equilibrium fields of T_0 and $\mathbf{w}_0$ equations (7)–(11) yield

$$[Ra_q\,\gamma - Ra_v\,\nabla(\mathbf{w}_0\mathbf{n})] \times \nabla T_0 = 0 \tag{15a}$$

$$\Delta T_0 = -1 \tag{15b}$$

$$\operatorname{div}\mathbf{w}_0 = 0 \tag{15c}$$

$$\operatorname{rot}\mathbf{w}_0 = \nabla T_0 \times \mathbf{n}. \tag{15d}$$

For the conditions of heating described above mechanical equilibrium is possible in which the functions T_0 and $\mathbf{w}_0$, satisfying the necessary boundary conditions, are as follows:

$$T_0 = \tfrac{1}{2}z(1-z) \tag{16a}$$

$$w_{0y} = w_{0z} = 0, \; w_{0x} \equiv w_0 = \tfrac{1}{12}(-1+6z-6z^2)\cos\alpha. \tag{16b}$$

Thus, in a mechanical quasi-equilibrium state the temperature distribution across the layer appears to be parabolic with the maximum value being in the middle of the layer. The pulsating velocity component is parallel to the layer boundaries. The temperature profile (16a) indicates that there is a potentially unstable stratification in the static gravity field in the upper half of the layer. On the other hand the longitudinal vibrational component, coupled with the transverse temperature gradient, produces conditions favourable for the occurrence of instability of vibrational convective nature [7, 8] (it should be stressed that the upper and lower halves of the layer must be considered on an equal basis in the sense of vibrational instability).

4. EQUILIBRIUM STABILITY

To study the stability of the equilibrium state, small disturbances of fields T_0, p_0 and $\mathbf{w}_0$ are considered

$$T = T_0 + T', \quad p = p_0 + p', \quad \mathbf{w} = \mathbf{w}_0 + \mathbf{w}'$$

as well as the associated mean flow with small velocity $\mathbf{v}'$. The substitution of the disturbed fields into the system of equations (7)–(11) and subsequent linearization yields a system of equations for disturbances

$$\frac{\partial \mathbf{v}'}{\partial t} = -\nabla p' + \Delta\mathbf{v}' + Ra_q\,T'\gamma + Ra_v\,[(\mathbf{w}_0\nabla)(T'\mathbf{n} - \mathbf{w}') + (\mathbf{w}'\nabla)(T_0\mathbf{n} - \mathbf{w}_0)] \tag{17}$$

$$Pr\frac{\partial T'}{\partial t} + \mathbf{v}'\nabla T_0 = \Delta T' \tag{18}$$

$$\operatorname{div}\mathbf{v}' = 0 \tag{19}$$

$$\operatorname{div}\mathbf{w}' = 0 \tag{20}$$

$$\operatorname{rot}\mathbf{w}' = \nabla T' \times \mathbf{n}. \tag{21}$$

The boundary conditions for disturbances resemble those presented by equations (12) and (13).

By analogy with the problem of vibrational convective fluid stability in a layer with different temperatures of bounding planes [8] it might be thought that the most dangerous disturbances are the two-dimensional disturbances $\mathbf{v}'(v'_x, 0, v'_z)$, $\mathbf{w}'(w'_x, 0, w'_z)$, T', p' which are independent of the coordinate y. For the amplitudes of normal disturbances of the form $\exp(-\lambda t + ikx)$ the following spectral problem can be obtained after having eliminated the pressure and horizontal components of the vectors $\mathbf{v}'$ and $\mathbf{w}'$:

$$-\lambda \mathrm{D}v = \mathrm{D}^2 v - k^2\,Ra_q\,\theta - Ra_v\,T'_0(ik\cos\alpha\,w' + k^2\sin\alpha\,w - k^2\cos^2\alpha\,\theta) \tag{22}$$

$$-\lambda\,Pr\,\theta = \mathrm{D}\theta - T'_0 v \tag{23}$$

$$\mathrm{D}w = ik\cos\alpha\,\theta' - k^2\sin\alpha\,\theta \tag{24}$$

$$z = 0, 1: \quad v = v' = 0, \quad \theta = 0, \quad w = 0. \tag{25}$$

Here v, w and θ are the disturbance amplitudes of the cross-wise components v_z, w_z and temperature which depend on z; the prime means differentiation with respect to z and the operator $\mathrm{D} = \mathrm{d}^2/\mathrm{d}z^2 - k^2$.

The spectral amplitude problem, equations (22)–(25), determines the eigenvalues—complex decrements $\lambda = \lambda_r + i\lambda_i$—as functions of all the other parameters Ra_q, Ra_v, Pr, α, and k. The stability limit can be found from the condition $\lambda_r = 0$. The imaginary part λ_i gives the frequency of oscillating disturbances and their phase velocity $c = \lambda_i/k$.†

† Recall that the analysis is carried out on the basis of an averaged system of equations of vibrational convection. Therefore, here and hereafter the discussion refers to slow oscillations the frequency of which is much smaller than the frequency of vibrations.

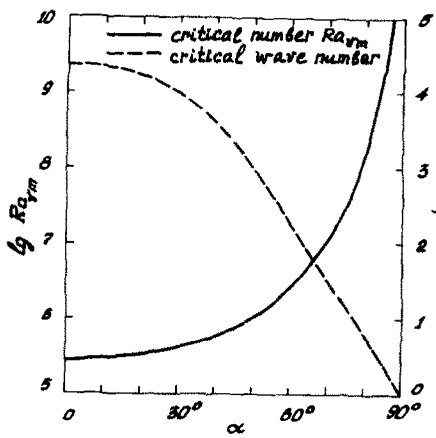

FIG. 2. Characteristics of vibrational instability at $Ra_q = 0$ vs the angle of inclination of the axis of vibration.

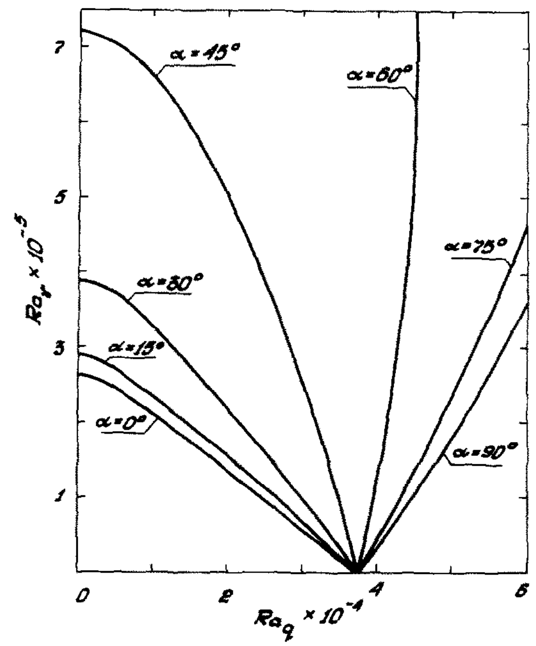

FIG. 3. Stability boundaries on the Ra_q–Ra_v plane at different values of α.

The boundary problem, equations (22)–(25), was integrated numerically by the Runge–Kutta–Merson method. Here, the main results of the solution will be presented (some data can be found in ref. [12] where equilibrium stability was also studied for another case of boundary conditions, i.e. when one of the layer boundaries is isothermal and the other is insulated).

First consider the limiting case $Ra_v = 0$ corresponding to the absence of vibration. In this case the instability is caused by monotonous disturbances ($\lambda_i = 0$) and is of thermo-gravitational nature. The critical Rayleigh number minimized with respect to the wave number is equal to $Ra_{qm} = 3.733 \times 10^4$ and is obtained at $k_m = 4.00$. These findings agree well with those drawn earlier in ref. [13].

Another limiting case corresponds to $Ra_q = 0$, i.e. to the absence of static gravity (zero gravity). Here the instability is of vibrational convective nature. The minimal critical value of the vibrational Rayleigh number depends on the angle α between the axis of vibration and the horizontal. As α increases, the critical number Ra_{vm} increases monotonically and tends to infinity as $\alpha \to 90°C$ (complete stabilization). The instability of this case ($Ra_q = 0$) is of monotonic character ($\lambda_i = 0$), whereas the critical wavelength increases without bound ($k_m \to 0$), with increase in α. Numerical results are presented in Fig. 2.

At arbitrary values of Ra_q and Ra_v the instability is caused by a combined action of static and vibrational mechanisms. The limits of equilibrium stability against dangerous disturbances on the plane Ra_q–Ra_v (the result of minimization with respect to k) for different directions of the vibration axis are presented in Fig. 3. The stability region is close to the origin of coordinates; the stability curves are symmetric about the Ra_v-axis.

At $\alpha = 0°$ and relatively small angles of the vibration axis to the horizontal (up to $\alpha = 52.6°$) both mechanisms of instability—static and vibrational—operate in a mutually destabilizing manner. In the limiting case of cross-wise oscillations ($\alpha = 90°$) the vibrational mechanism of instability is not operative. The effect of vibration appears here to be purely stabilizing, with the critical Rayleigh number Ra_{qm} increasing with Ra_v according to the law $Ra_{qm} = 79.5 \times Ra_v^{1/2}$ at high values of the latter parameter. It follows from here that there exists a certain limiting value of the vibration 'intensivity' parameter $(b\Omega)_* = 0.01779 gh^2/\sqrt{(\nu\chi)}$ at which complete stabilization can be achieved: if $(b\Omega) > (b\Omega)_*$, the equilibrium is stable at any power of heat release Q.

Figure 4(a) shows the dependence of the critical values of the wave number k_m of most dangerous disturbances on Ra_q. The branches of the curves $k_m(Ra_q)$ correspond to the neutral curves in Fig. 3.

Calculations show that in contrast to the case of a horizontal layer with the boundaries of different temperatures, the instability is, generally speaking, of oscillating character. The monotonic instability develops only in the limiting cases of longitudinal ($\alpha = 0°$) and transverse ($\alpha = 90°$) vibrations at all of the angles α in the above-mentioned cases of $Ra_v = 0$ and $Ra_q = 0$. Generally the instability arises due to the oscillating disturbances in the form of rolls that drift in the direction opposite to the x-axis with the phase velocity $c = \lambda_i/k$. Figure 4(b) shows the dependence of the phase velocity of most dangerous disturbances c_m on Ra_q.

Generally speaking, in the case of oscillating disturbances the critical parameters depend on the Prandtl number. However, calculations carried out within the range $0.1 \leqslant Pr \leqslant 10$ show that the stability limits (Fig. 3) and the critical wave numbers (Fig. 4(a)) are virtually independent of Pr. As for the phase velocity of neutral disturbances, it depends on Pr but

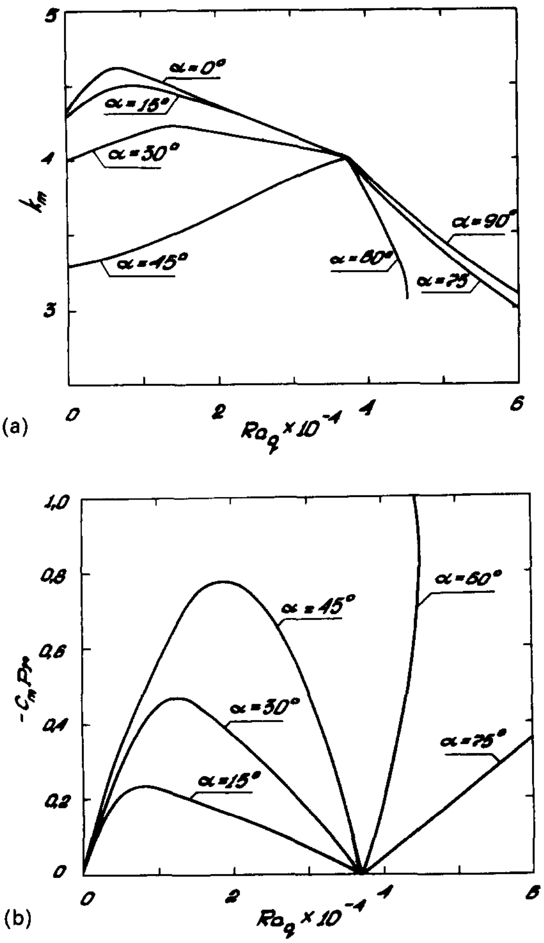

FIG. 4. Characteristics of critical disturbances at different values of α.

the product $c_m \cdot Pr$ (Fig. 4(b)) does not practically depend on Pr at least within the range mentioned.

The form of critical disturbances is defined by the eigenfunctions of the spectral problem, equations (22)–(25). Figure 5 presents the profiles of the cross-wise velocity component v_z (the normalization is conditional) which correspond to the most dangerous neutral disturbances for the case of longitudinal vibrations ($\alpha = 0°$). At small values of Ra_v, when the static instability mechanism presents, in the upper (unstably stratified) part of the layer there arises a single vortex disturbance which penetrates into the lower part of the layer where there is stable stratification. With an increase of Ra_v (and, consequently, decrease of Ra_q) the nature of instability gradually changes and at small values of Ra_q the instability acquires a vibrational character. Naturally, this is reflected in the form of disturbance which becomes two-vortex over the vertical.

In general this situation persists also at $\alpha \neq 0°$. However, the structure becomes somewhat complicated due to the drift of critical disturbances. They lose their symmetry but the general tendency of transition from one-vortex structure in the region of small Ra_v to the two-vortex one in the region of small Ra_q remains.

5. FINITE-AMPLITUDE REGIMES: FORMULATION OF THE PROBLEM

The linear theory based on the concept of small disturbances allows one to determine the stability limit of the mechanical equilibrium. To study the finite-amplitude regimes developing due to the loss of stability, it is necessary to obtain the solution to the full non-linear system of equations (7)–(11). Further a two-dimensional flow in the x–z plane will be considered which possesses periodicity along the x-axis.

The stream functions ψ and F for two-dimensional fields $\mathbf{v}$ and $\mathbf{w}$ are introduced according to the following relations:

$$v_x = -\frac{\partial \psi}{\partial z}, \quad v_z = \frac{\partial \psi}{\partial x} \tag{26a}$$

$$w_x = -\frac{\partial F}{\partial z}, \quad w_z = \frac{\partial F}{\partial x}. \tag{26b}$$

By eliminating the pressure and potential φ, equations (7)–(11) will give a system of equations of the averaged vibrational convection for the variables ψ, T, and F

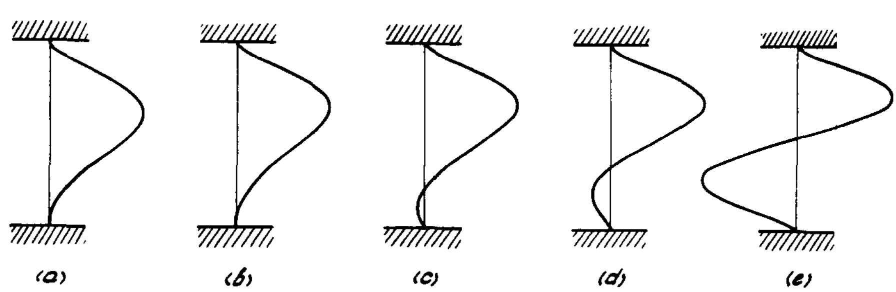

FIG. 5. Profiles of the transverse velocity component on the stability boundary in the case of longitudinal vibration: (a) $Ra_q = 3.733 \times 10^4$, $Ra_v = 0$; (b) $Ra_q = 1 \times 10^4$, $Ra_v = 2.11 \times 10^5$; (c) $Ra_q = 4 \times 10^3$, $Ra_v = 2.53 \times 10^5$; (d) $Ra_q = 2 \times 10^3$, $Ra_v = 2.64 \times 10^5$; (e) $Ra_q = 0$, $Ra_v = 2.69 \times 10^5$.

$$\frac{\partial \Delta\psi}{\partial t}+\frac{1}{Pr}\left(\frac{\partial \psi}{\partial x}\frac{\partial \Delta\psi}{\partial z}-\frac{\partial \psi}{\partial z}\frac{\partial \Delta\psi}{\partial x}\right)=\Delta\Delta\psi+Ra_q\frac{\partial T}{\partial x}$$
$$+Ra_v\left[\left(\frac{\partial^2 F}{\partial x^2}\frac{\partial T}{\partial z}-\frac{\partial^2 F}{\partial x\,\partial z}\frac{\partial T}{\partial x}\right)\sin\alpha\right.$$
$$\left.-\left(\frac{\partial^2 F}{\partial x\,\partial z}\frac{\partial T}{\partial z}-\frac{\partial^2 F}{\partial z^2}\frac{\partial T}{\partial x}\right)\cos\alpha\right] \quad (27)$$

$$Pr\frac{\partial T}{\partial t}+\left(\frac{\partial \psi}{\partial x}\frac{\partial T}{\partial z}-\frac{\partial \psi}{\partial z}\frac{\partial T}{\partial x}\right)=\Delta T+1 \quad (28)$$

$$\Delta F=\frac{\partial T}{\partial x}\sin\alpha-\frac{\partial T}{\partial z}\cos\alpha. \quad (29)$$

Here $\Delta=\partial^2/\partial x^2+\partial^2/\partial z^2$ is the two-dimensional Laplacian.

The no-slip, isothermicity and non-overflow conditions for the pulsating velocity component are imposed on the horizontal boundaries of the layer

$$z=0,1:\quad \psi=0,\quad \frac{\partial\psi}{\partial z}=0,\quad T=0,\quad F=0. \quad (30)$$

The solution of the non-linear system, equations (27)–(29), is sought in the rectangular region $0\leqslant x\leqslant L$, $0\leqslant z\leqslant 1$, with the periodicity conditions being imposed on the side (vertical) boundaries

$$f(0,z)=f(L,z) \quad (31)$$

where L is the spatial period and f is any of the fields that describe fluid flow.

The problem was solved by the method of finite differences. The 'two-field' method was used, with the vorticity introduced as an additional variable. An explicit finite-difference scheme was used as well as an implicit scheme of the method of fractional steps. The Poisson equations were iterated by the method of successive upper relaxation. To perform the finite-difference approximation of the boundary conditions for vorticity Tom's formula was used. In calculations the Prandtl number was taken to be equal to 1. The spatial period of solution was also fixed: $L=1.5$. To this value there corresponds the wave number $k=2\pi/L=4.19$ which is close to the critical wave number k_m of the most dangerous disturbances at least at not too large inclination angles α of the vibration axis (Fig. 4(a)). The main calculations were carried out on a 31×21 grid; control calculations on a 46×31 grid showed that the error in calculating the integral characteristics does not exceed (in the investigated range of parameters) several per cent.

6. FINITE-AMPLITUDE REGIMES: RESULTS

First the case of the longitudinal orientation of the vibrational axis $\alpha=0°$ is considered. As has already been mentioned above (Section 4) in this case, as soon as the parameter Ra_v (or Ra_q) reaches its critical value, the mechanical equilibrium loses its stability against

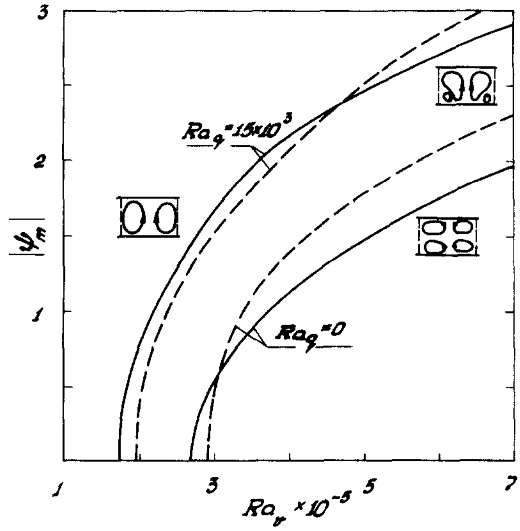

FIG. 6. Maximum stream function vs Ra_v for two values of Ra_q. ——, $\alpha=0°$; – – –, $\alpha=15°$.

the disturbances that monotonically vary in time ($\lambda_i=0$).

Calculations show that in the region of parameters Ra_q, Ra_v that correspond to the stable equilibrium all of the initial disturbances damp. Disturbances develop in the supercritical region, and, as a result of the transitional process, the secondary stationary regime of convection is established the form and intensity of which depend on the governing parameters Ra_q and Ra_v. The characteristic of the convective flow rate is taken to be the maximum value (in magnitude) of the stream function $|\psi_m|$ which determines circulation in the zone of the most intensive vortex.

The dependence of $|\psi_m|$ on Ra_v for two values of Ra_q is shown in Fig. 6. The value $Ra_q=0$ corresponds to zero gravity. In this case convection is induced by the vibrational mechanism and is of symmetrical four-vortex shape. Extrapolation of the dependence of $|\psi_m|$ on Ra_v to the value $|\psi_m|=0$ makes it possible to determine the critical point which agrees well with the results of the linear theory of stability. Above the threshold the quantity $|\psi_m|$ increases monotonically with an increasing supercriticality. Near the critical point the asymptotic law $|\psi_m|\sim(Ra_v-Ra_{v*})^{1/2}$ takes place.

The value $Ra_q=1.5\times10^4$ conforms already to rather a high Rayleigh number of gravity. For this reason a two-vortex flow of mainly gravitational nature is induced on the stability boundary. However, as Ra_v increases, the relative role of the vibrational convective mechanism increases and thus the evolutionary reconstruction of the flow shape takes place. It gradually achieves the four-vortex structure characteristic of vibrational convection. Just as at $Ra_q=0$ the root-square law is valid near the bifurcation point and this testifies to the 'soft' regime of convection excitation.

The Nusselt number is introduced as a charac-

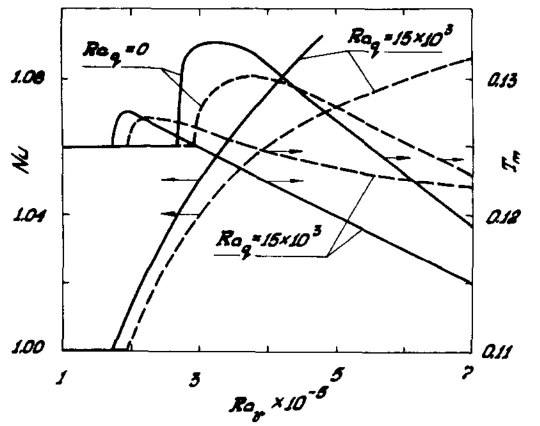

FIG. 7. Nusselt number Nu and maximum temperature T_m vs Ra_v for two values of Ra_q. ——, $\alpha = 0°$; – – –, $\alpha = 15°$.

teristic of heat transfer. It is defined in terms of the dimensionless temperature field as

$$Nu = \int_0^L \left(\frac{\partial T}{\partial x}\right)_{z=1} dx \bigg/ \int_0^L \left(\frac{\partial T_0}{\partial z}\right)_{z=1} dx.$$

It is evident that the Nusselt number represents a heat flux through the upper layer boundary over the stretch of length L normalized to its value in the pure thermal diffusion regime (i.e. in equilibrium). In equilibrium $Nu = 1$. In the case of purely vibrational convection ($Ra_q = 0$), with the temperature field being symmetric, $Nu = 1$ just as at equilibrium. However, when $Ra_q \neq 0$ the gravitational component of convection induces the temperature field asymmetry as a result of which the heat flux through the upper boundary increases by a certain value and the heat flux through the lower boundary decreases by the same value. Thus, the difference $Nu-1$ at $Ra_q \neq 0$ characterizes the heat flux through the layer caused by the gravitational convective component.

Figure 7 shows the dependence of Nu on Ra_v. When $Ra_q = 0$, $Nu = 1$ and when $Ra_q = 1.5 \times 10^4$ there occurs an overall upward heat flux through the layer. Its magnitude increases with the growth of supercriticality; near the bifurcation point $Nu-1 \sim (Ra_v - Ra_{v*})$.

An interesting characteristic of the temperature field is the maximum value of temperature T_m in the computational domain. As is seen from equation (16a), $T_m = 0.125$ in equilibrium. When convection excited at the critical point, the value of T_m first slightly increases with the growth of Ra_v and then decreases.

Calculations of averaged velocity and temperature fields and also of integral flow characteristics at two fixed values $Ra_v = 3.5 \times 10^5$ and 7×10^5 also reveal a gradual transition from four-vortex vibrational to two-vortex thermogravitational convection as Ra_q increases. A new feature here is the occurrence of the regime of stationary oscillations on the attainment of some value of Ra_q dependent on Ra_v (for

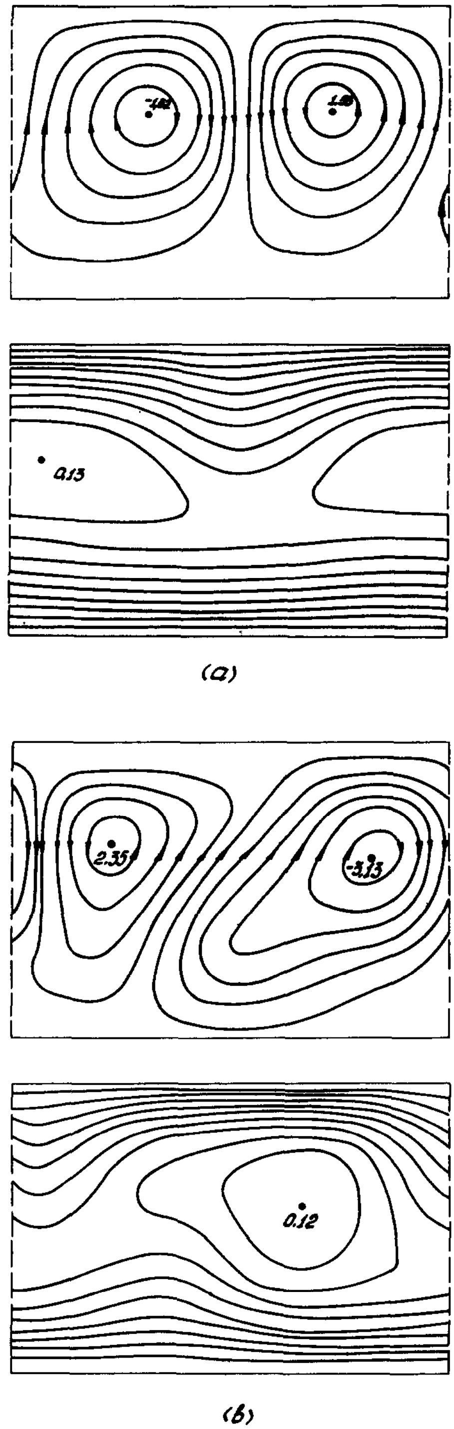

FIG. 8. Stream lines and isotherms of drifting vortical structures at $\alpha = 15°$: (a) negative velocity U, $Ra_q = 1.5 \times 10^4$, $Ra_v = 3 \times 10^5$; (b) positive velocity U, $Ra_q = 1.5 \times 10^4$, $Ra_v = 7 \times 10^5$.

$Ra_v = 3.5 \times 10^5$ and 7×10^5 these values are equal to 1×10^5 and 2×10^4, respectively).

The oscillations involve regular reconstructions of the vortex flow pattern and change of the circulation sign in each cell and are not accompanied by vortex drift along the layer. It should be noted that there

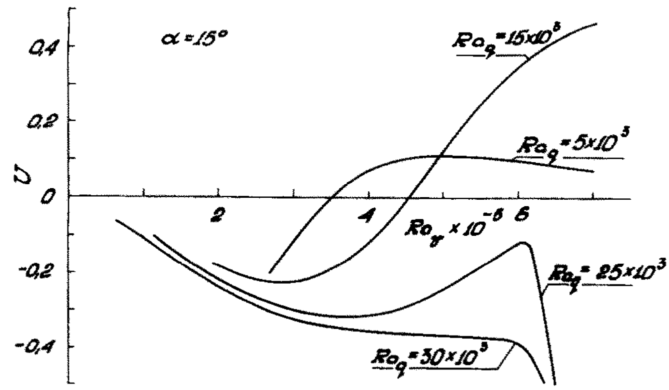

FIG. 9. Drift velocity U vs Ra_v for four values of Ra_q at $\alpha = 15°$.

oscillations are not associated with the vibrational mechanism of convection; they occur also in the case of a static gravity field ($Ra_v = 0$) which was studied earlier [14].

Now, the influence of the vibration axis direction on the supercritical regimes of convection will be considered.

When the vibration axis is moderately inclined to the horizontal ($\alpha = 15°$), the behaviour of the bifurcational characteristics $|\psi_m|$, $Nu-1$ and T_m is close to that obtained for $\alpha = 0°$ (Figs. 6 and 7). The essential difference from the case $\alpha = 0°$ consists in the fact that the equilibrium instability is now caused by oscillating disturbances that possess a negative phase velocity. The finite-amplitude regime developing above the threshold is a periodical, with respect to x, vortical system drifting to the left with some velocity U. The drift velocity determined numerically, on the stability boundary agrees well with the phase velocity of the neutral disturbance in accordance with the linear theory of stability. Figure 8(a) shows an example of the instantaneous stream function and temperature fields corresponding to the vortex system drifting to the left.

The drift velocity U in the supercritical region depends on the Rayleigh numbers Ra_q and Ra_v. It is important to observe that at certain values of the above parameters the drift direction can change to the opposite direction.

Figure 9 shows the dependence of velocity U on Ra_v for some values of Ra_q. It is seen that at $Ra_q = 5\times10^3$ and 1.5×10^4 the velocity changes its sign with an increase in Ra_v whereas at $Ra_q = 2.5\times10^4$ and 3×10^4 the velocity is negative over the entire region. An example of instantaneous fields that corresponds to the regime drifting to the right is given in Fig. 8(b). The treatment of plots similar to Fig. 9 allows one to construct a map of regimes on the Ra_q–Ra_v plane where the regions of equilibrium and of finite-amplitude regimes with negative and positive values of drift velocity are depicted (Fig. 10). The boundary of the regions $U < 0$ and $U > 0$ is the line along which the stationary regimes of finite-amplitude convection are realized.

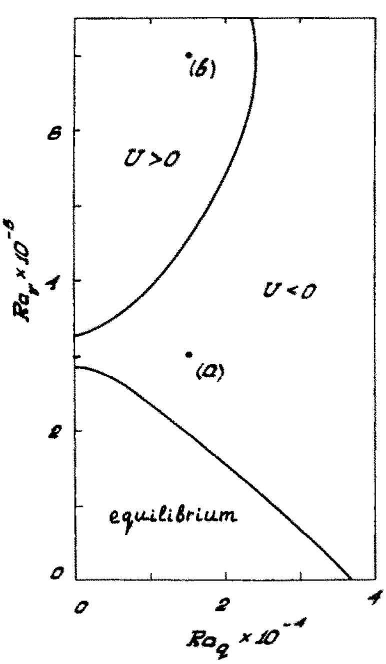

FIG. 10. Map of regimes for $\alpha = 15°$. The regions of equilibrium and of finite-amplitude regimes with negative and positive drift velocities are shown. Points (a) and (b) correspond to the instantaneous fields depicted in Fig. 9.

As the angle of the vibration axis inclined to the horizontal increases there occurs the displacement of the boundary of regimes with negative and positive drift velocity and the region with $U > 0$ ultimately disappears. Figure 11 presents the dependence of U on Ra_v at fixed Ra_q for different angles α. It is seen that at the given Ra_q and $\alpha = 20°$ the waves with the positive drift direction exist only in a narrow range of Ra_v, whereas at $\alpha = 45°$ this range is absent. An interesting feature is that at $\alpha = 20°$ near the point $Ra_v = 7\times10^5$ nonsingularity exists. In a very narrow range of Ra_v, for a given value of Ra_v there corresponds three possible non-linear wave regimes, with one of them being unstable. The transition is accompanied by wave regime reconstruction and hysteresis.

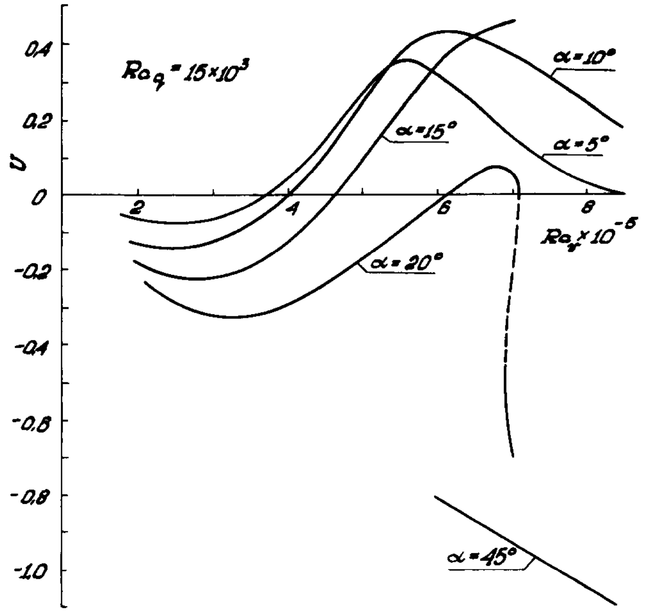

FIG. 11. Drift velocity U vs Ra_v at different angles α of inclination for $Ra_q = 1.5\times10^4$. The dashed portion of the curve $\alpha = 20°$ indicates the region of unstable wave regimes.

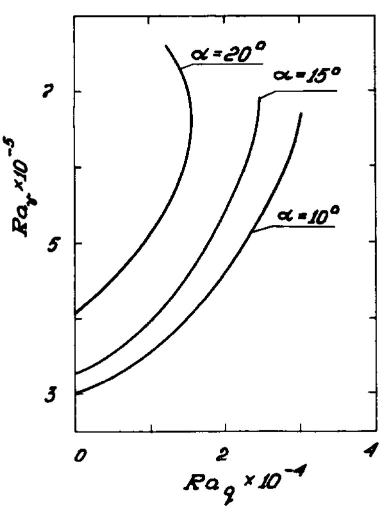

FIG. 12. Lines of stationary finite-amplitude regimes for three orientations of vibrations.

Finally, Fig. 12 presents the curves of stationary regimes (lines of the change in the sign of U) on the Ra_q–Ra_v plane for three values of the angle α. These curves demonstrate the displacement of the regions with $U > 0$ as the value of α increases.

7. CONCLUSIONS

The consideration of the new problem in the theory of vibrational convection in a plane horizontal fluid layer with uniform distributed heat sources showed the possibility of the state of mechanic quasi-equilibrium in the case of isothermal layer boundaries. It loses its stability with an increase of the characteristic parameters: gravitational and vibrational Rayleigh numbers. Depending on the values of the governing parameters, the instability may have a thermo-gravitational or mixed nature. In the case of the axis of vibration being inclined to the horizontal, the instability is associated with the development of wave disturbances. The study of non-linear disturbances shows that, depending on the values of the parameters, the loss of equilibrium stability results in the finite-amplitude regimes of stationary or wave type, with the wave regimes occurring with drift velocities that differ in magnitude and sign. The finite-amplitude vibrational convection leads to the rise of upward heat flux across the layer. The magnitude of this flux is determined by Rayleigh numbers and by the direction of the axis of vibration.

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CONVECTION VIBRATIONNELLE DANS UNE COUCHE FLUIDE HORIZONTALE, AVEC DES SOURCES DE CHALEUR INTERNES

Résumé—Un examen théorique de la convection d'un fluide avec des sources de chaleur internes uniformes dans une couche horizontale plane est faite pour une vibration harmonique de grande fréquence avec orientation arbitraire. Les frontières solides de la couche sont à des températures identiques. Le problème linéaire de la stabilité mécanique est résolu numériquement. Les limites de stabilité sont déterminées ainsi que les paramètres et la forme des perturbations critiques. Les régimes non linéaires, d'amplitude finie, de transfert thermique sont étudiés par la méthode de différences finies. La présence des régimes stationnaires et d'ondes est établie et leurs caractéristiques sont étudiées.

VIBRATIONSKONVEKTION IN EINER HORIZONTALEN FLUIDSCHICHT MIT INNERN WÄRMEQUELLEN

Zusammenfassung—Die Konvektion in einer ebenen horizontalen Fluidschicht mit gleichförmig verteilten inneren Wärmequellen wird unter dem Einfluß harmonischer hochfrequenter Schwingungen beliebiger Orientierung theoretisch untersucht. Die festen Begrenzungen der Schicht werden auf gleichförmiger Temperatur gehalten. Das lineare Problem der mechanischen Gleichgewichtsstabilität wird numerisch gelöst. Die Stabilitätsgrenzen, Parameter und die Form kritischen Störungen werden ermittelt. Die nichtlinearen Bereiche begrenzter Amplitude von Konvektion und Wärmeübergang werden mit der Methode der finiten Differenzen untersucht. Die Existenz stationärer und driftender Bereiche wird nachgewiesen, deren Eigenschaften werden untersucht.

ВИБРАЦИОННАЯ КОНВЕКЦИЯ В ГОРИЗОНТАЛЬНОМ СЛОЕ ЖИДКОСТИ С ВНУТРЕННИМИ ИСТОЧНИКАМИ ТЕПЛА

Аннотация—Теоретически исследуется конвекция жидкости с однородными внутренними источниками тепла в плоском горизонтальном слое при наличии гармонической высокочастотной вибрации произвольного направления. Твердые границы слоя поддерживаются при одинаковых температурах. Численно решена линейная задача устойчивости механического равновесия. Определены границы устойчивости, параметры и форма критических возмущений. Методом конечных разностей исследованы нелинейные конечно-амплитудные режимы конвекции и теплопереноса. Установлено наличие стационарных и волновых (дрейфующих) режимов. Изучены их характеристики.